$\vec{0} = \langle 0,0,0 \rangle$

$\|\vec{0}\| = 0$

IF $\|\vec{u}\| = 0$, THEN $\vec{u} = \vec{0}$

UNIT VECTORS

$\vec{v}$ IS A UNIT VECTOR IF AND ONLY IF $\|\vec{v}\| = 1$

IS $\langle \frac{1}{3}, \frac{1}{3}, \frac{1}{3} \rangle$ A UNIT VECTOR? $\boxed{\text{NO}}$

$$\left\|\left\langle \tfrac{1}{3}, \tfrac{1}{3}, \tfrac{1}{3} \right\rangle\right\| = \sqrt{\left(\tfrac{1}{3}\right)^2 + \left(\tfrac{1}{3}\right)^2 + \left(\tfrac{1}{3}\right)^2} = \sqrt{\frac{3}{9}} = \frac{\sqrt{3}}{3} \neq 1$$

IF $\vec{v}$ IS A UNIT VECTOR, $2\vec{v}$ IS IN THE SAME DIR AS $\vec{v}$

$\|2\vec{v}\| = |2|\,\|\vec{v}\| = 2 \cdot 1 = 2$

$-2\vec{v}$ IS IN THE OPPOSITE DIR AS $\vec{v}$

WITH MAGNITUDE 2

IF $\vec{v} = \langle v_1, v_2, v_3 \rangle$

THEN FIND $\underbrace{\text{A UNIT VECTOR}}_{\vec{u}}$ IN THE <u>SAME</u> DIR AS $\vec{v}$.

$$\|\vec{u}\| = 1$$

$$\vec{u} = k\vec{v}, \quad k > 0$$

$$\|k\vec{v}\| = 1$$

$$|k|\,\|\vec{v}\| = 1$$

$$k\|\vec{v}\| = 1$$

$$k = \frac{1}{\|\vec{v}\|}$$

$$\vec{u} = \frac{1}{\|\vec{v}\|}\vec{v} = \frac{\vec{v}}{\|\vec{v}\|}$$

eg. IF $\vec{v} = \langle \frac{1}{3}, \frac{1}{3}, \frac{1}{3} \rangle$

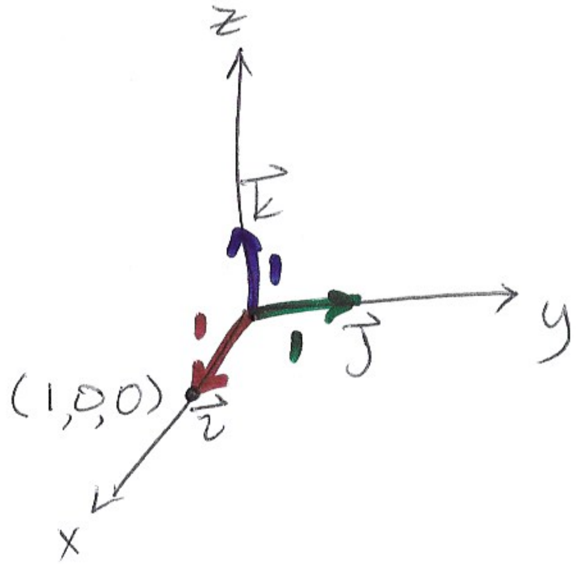
THEN $\vec{u} = \dfrac{\vec{v}}{\|\vec{v}\|} = \dfrac{\langle \frac{1}{3}, \frac{1}{3}, \frac{1}{3} \rangle}{\frac{\sqrt{3}}{3}}$

$$= \frac{3}{\sqrt{3}}\langle \tfrac{1}{3}, \tfrac{1}{3}, \tfrac{1}{3} \rangle$$

$= \langle \frac{1}{\sqrt{3}}, \frac{1}{\sqrt{3}}, \frac{1}{\sqrt{3}} \rangle$ IS ~~A~~ THE UNIT VECTOR IN THE SAME DIR AS $\vec{v}$

FIND A VECTOR OF MAGNITUDE 6 IN THE OPPOSITE DIRECTION AS $\vec{v} = \left\langle \frac{1}{3}, \frac{1}{3}, \frac{1}{3} \right\rangle$

$$-6\vec{u} = -6\left\langle \frac{1}{\sqrt{3}}, \frac{1}{\sqrt{3}}, \frac{1}{\sqrt{3}} \right\rangle = \langle -2\sqrt{3}, -2\sqrt{3}, -2\sqrt{3} \rangle$$



$\vec{i} = \langle 1, 0, 0 \rangle$

$\vec{j} = \langle 0, 1, 0 \rangle$

$\vec{k} = \langle 0, 0, 1 \rangle$

IF $\vec{v} = \langle a, b, c \rangle$

THEN $\vec{v} = \langle a, 0, 0 \rangle + \langle 0, b, 0 \rangle + \langle 0, 0, c \rangle$

$= a\langle 1, 0, 0 \rangle + b\langle 0, 1, 0 \rangle + c\langle 0, 0, 1 \rangle$

$= a\vec{i} + b\vec{j} + c\vec{k}$ ← LINEAR COMBINATION OF $\vec{i}, \vec{j}, \vec{k}$

$a\vec{i} + b\vec{j} + c\vec{k} = \langle a, b, c \rangle$

eg. $3\vec{k} - 2\vec{i} = \langle -2, 0, 3 \rangle$

12.3 DOT PRODUCT / SCALAR PRODUCT

$\vec{U} \cdot \vec{V} = U_1 V_1 + U_2 V_2 + U_3 V_3$

ALGEBRAIC DEFINITION OF DOT PRODUCT

$\vec{U} = \langle U_1, U_2, U_3 \rangle$

$\vec{V} = \langle V_1, V_2, V_3 \rangle$

eg. $\langle 2,-1,3 \rangle \cdot \langle -5,1,-1 \rangle = 2(-5) + (-1)1 + 3(-1)$

$= -10 - 1 - 3$

$= -14$

GEOMETRIC DEFINITION OF DOT PRODUCT

$\vec{U} \cdot \vec{V} = \|\vec{U}\| \, \|\vec{V}\| \cos\theta$

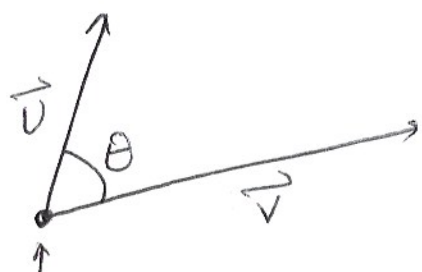
NOTE:

$\vec{U} \cdot \vec{V} > 0 \rightarrow \cos\theta > 0$

IE. ANGLE θ BETWEEN $\vec{U}, \vec{V}$ IS ACUTE

$\vec{U} \cdot \vec{V} < 0 \rightarrow \cos\theta < 0$

IE. θ IS OBTUSE



SAME INITIAL POINT

θ = SMALLER ANGLE CREATED BY $\vec{U}, \vec{V}$ WHEN THEY SHARE AN INITIAL POINT

IF $\vec{U} \cdot \vec{V} = 0$, THEN $\vec{U} \perp \vec{V}$ OR $\vec{U} = \vec{0}$ OR $\vec{V} = \vec{0}$

IF $\vec{U} \cdot \vec{V} = 0$, WE SAY $\vec{U}, \vec{V}$ ARE ORTHOGONAL

IF $\vec{U}, \vec{V}$ ARE ORTHOGONAL AND BOTH ARE NOT $\vec{0}$, THEN $\vec{U} \perp \vec{V}$

IF $\vec{u} = \langle 3,-2,1\rangle$ AND $\vec{v} = \langle -2,-1,c\rangle$ AND $\vec{u}\perp\vec{v}$, FIND C

$$\vec{u}\perp\vec{v} \longrightarrow \vec{u}\cdot\vec{v}=0$$

$$\langle 3,-2,1\rangle\cdot\langle -2,-1,c\rangle = 0$$

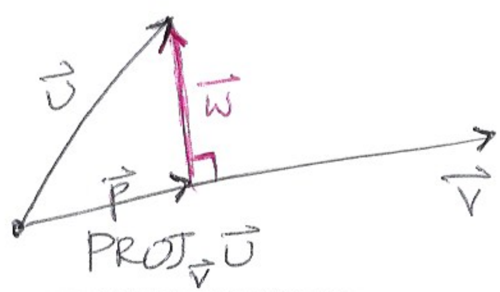
$$-6+2+c=0$$

$$c=4$$

PROJECTIONS (≈ SHADOW)

VECTOR PROJECTION OF $\vec{u}$ ONTO $\vec{v}$

$$= \text{PROJ}_{\vec{v}}\vec{u} = \frac{\vec{v}\cdot\vec{u}}{\vec{v}\cdot\vec{v}}\vec{v}$$



$$\vec{p}=k\vec{v}$$

$$\vec{p}\cdot\vec{w}=0 \quad (\vec{p}\perp\vec{w})$$

$$\vec{p}+\vec{w}=\vec{u}$$

$$k\vec{v}\cdot\vec{w}=0 \longrightarrow k\vec{v}\cdot(\vec{u}-k\vec{v})=0$$

$$k\vec{v}+\vec{w}=\vec{u} \longrightarrow \vec{w}=\vec{u}-k\vec{v}$$

$$\vec{v}\cdot(\vec{u}-k\vec{v})=0$$

$$\vec{v}\cdot\vec{u}-\vec{v}\cdot k\vec{v}=0$$

$$\vec{v}\cdot\vec{u}-k(\vec{v}\cdot\vec{v})=0$$

$$k=\frac{\vec{v}\cdot\vec{u}}{\vec{v}\cdot\vec{v}} \longrightarrow \vec{p}=\frac{\vec{v}\cdot\vec{u}}{\vec{v}\cdot\vec{v}}\vec{v}$$

PROPERTIES

$$\vec{u}\cdot\vec{v}=\vec{v}\cdot\vec{u}$$

$$k(\vec{u}\cdot\vec{v})=k\vec{u}\cdot\vec{v}=\vec{u}\cdot k\vec{v}$$

$$(\vec{u}+\vec{v})\cdot\vec{w}=\vec{u}\cdot\vec{w}+\vec{v}\cdot\vec{w}$$